Mass Defect and Binding Energy

Objectives

After going through the module, learner will be able to

- Understand the equivalence of mass and energy
- Calculate the mass defect
- Appreciate the decrease in mass when a nucleus is formed from its constituents
- Explain the release of energy in the process of fusion and fission
- Correlate binding energy and binding energy per nucleon
- Understand and interpret the binding energy per nucleon vs atomic number graph

Content Outline

- Unit Syllabus
- Module wise distribution of unit syllabus
- Words you must know
- Introduction
- Mass and energy, Einstein's equation $E = mc^2$
- Mass defect
- MeV
- Nuclear binding energy
- Understanding the graph and interpretations from it
- Summary

Unit Syllabus

Unit 8

Atoms and Nuclei

Chapter 12 Atoms

Alpha particle scattering experiment, Rutherford's model of atom, Bohr model, energy levels, hydrogen spectrum

Chapter 13 Nuclei

Composition and size of nucleus, radioactivity, alpha, beta and gamma particles/rays and their properties, radioactive decay laws

Mass energy relations, mass defect, binding energy per nucleon and its variation with mass number, nuclear fission and nuclear fusion

Module Wise Distribution Of Unit Syllabus -7 Modules

Module 1	● Introduction ● Early models of atom ● Alpha particle scattering and Rutherford's Nuclear model of atom ● Alpha particle trajectory ● Results and interpretations ● Size of nucleus ● What Rutherford's model could not explain
Module 2	● Bohr's model of hydrogen atom ● Bohr's postulates ● Electron orbits, what do they look like? ● Radius of Bohr orbits ● Energy levels, Energy states, energy unit eV ● Lowest energy -13.6 eV interpretation ● Velocity of electrons in orbits
Module 3	● The line Spectrum of hydrogen atom ● de Broglie's explanation of Bohr 's second postulate of quantisation ● Departures from Bohr model energy bands ● Pauli's Exclusion Principle and Heisenberg's uncertainty principle leading to energy bands
Module 4	● Atomic masses and composition of nucleus ● Discovery of neutron ● Size of nucleus ● Nuclear forces ● Energy levels inside the nucleus

Module 5	● Mass and energy, Einstein's equation $E = mc^2$ ● Mass defect ● MeV ● Nuclear binding energy ● Binding energy per nucleon as a function of mass number ● Understanding the graph and interpretations from it
Module 6	● Radioactivity ● Laws of radioactivity ● Half life ● Rate of decay -disintegration constant ● Alpha decay ● Beta decay ● Gamma decay
Module 7	● Nuclear energy ● Fission ● Controlled fission reaction ● Nuclear Reactor ● India's atomic energy programme ● Nuclear Fusion – energy generation in stars ● Controlled thermonuclear fusion

Module 5

Words You Must Know

Let us remember the words and the concepts which we are familiar

Atom structure is an atom the smallest independent entity of an element .It consists of a small, central, massive and positive core surrounded by orbiting electrons nucleus. The central positive core of the atom is called the nucleus. size of an atom The size of an atom is of the order of 10^{-10}m, the size of a nucleus is of the order of 10^{-15} m nucleons. Nucleus is made up of neutrons and protons, they being the constituents of a nucleus are also called nucleons.

Mass number: If a nucleus has **Z number of protons** and **N number of neutrons**, then its mass number A.

$$A = Z + N$$

A nucleus of an element is represented as $_Z X^A$

Isotopes: The atoms of an element, which have the same atomic number (Z), but different mass number (A), are called isotopes. For example $_1H^1$, $_1H^2$ and $_1H^3$ are isotopes. They have the same number of protons but different numbers of neutrons.

Isobars: The atoms, which have the same mass number (A) , but different atomic number (Z), are called isobars. They are the atoms of different elements, for example, $_1H^3$, $_2He^3$ are isobars.

Isotones: The atoms, whose nuclei have, same number of neutrons, are called isotones.

Nuclear size: The radius R , of a nucleus having mass number A is given by the expression

$$R = R_0 A^{1/3}, \text{ where } R_0 = 1.1 \times 10^{-15} \text{ m}$$

Nuclear density: Nuclear density = (mass of the nucleus/ volume of the nucleus) Nuclear density is independent of mass number.

Properties of a neutron: A neutron is a neutral particle carrying no charge, and having mass slightly more than that of a proton. A neutron is stable inside the nucleus, but a free neutron is unstable and has a mean life of 1000 second.

Nuclear forces: In spite of a Columbian repulsive force between protons, the nucleons stay inside a nucleus because of a strong attractive force called nuclear force.

- Nuclear forces are the strongest forces in nature
- Nuclear forces are short range forces
- Nuclear forces are saturated forces
- Nuclear forces are charge independent
- Nuclear forces are spin dependent, non-central forces.

Atomic mass unit: One atomic mass unit is defined as (1/12)th of the mass of one $_6C^{12}$ atom.

1 atomic mass unit = 1.66×10^{-27} kg

eV energy gained by an electron when subjected to a potential difference of one volt.

Introduction

From the experiment on scattering of alpha particles we have learnt that in every atom, entire positive charge and almost entire mass are densely concentrated at the centre of the atom forming its nucleus. Protons and neutrons are the constituents of a nucleus. Alpha particle scattering experiment also showed that the radius of a nucleus is smaller than the radius of an atom by a factor of about 10^4. This means the volume of a nucleus is about 10 -12 times the volume of the atom. The constituents of atoms called the nucleons are held together in a very small volume by nuclear force. As a nucleus consists of neutrons and protons, therefore it may be expected that the mass of the nucleus is equal to the total mass of its protons and neutrons. The question is, is it really the case?

In this module we will deal with what keeps the nucleus together, the energy associated with the nucleus.

Mass and Energy (Einstein's Mass - Energy) Relation

From his theory of relativity, Einstein showed that it is necessary to treat mass as another form of energy. Einstein showed that like any other form of energy, mass – energy can also be converted into other forms of energy, say kinetic energy and vice-versa.

Einstein's famous mass –energy equivalence relation is

$E = mc^2$

The energy equivalent of mass m is given by the above equation; here c is the velocity of light in vacuum.

Example

Calculate the energy equivalent of 1g of substance.

Solution

Using the relation $E = mc^2$

$E= 10^{-3}\ \text{kg} \times (3\times10^{8})^2 \text{m/s}^2 = \mathbf{9 \times 10^{13}\ J}$

Thus we see that if 1g of a matter is converted to energy, there is a release of enormous amounts of energy.

Experimental verification of Einstein's mass energy relation has been achieved in the study of nuclear reactions amongst nucleons.

This concept is important in nuclear masses and the interaction of nuclei with one another.

In classical mechanics, the laws of conservation of mass and energy are two separate principles independent of each other. The relation $E = mc^2$, leads to the unification of the two laws into one law.

For nuclear force and mass defect watch

Example

Calculate the energy equivalence of 1 atomic mass unit in MeV.

Solution

$m = 1u = 1.66\times10^{-27}$ kg

$E = mc^2$

$E = 1.66\times10^{-27}\ \text{kg} \times(3\times10^8\ \text{m/s})^2$

$E = 1.49\times 10^{-10}$ J

$= (1.49\times10^{-10}/1.6\times10^{-13})$ MeV $= 931.25$ MeV

Mass Defect

Nucleus of an atom is generally stable. As the protons being positively charged would exert a very strong force of repulsion on each other, so let us try to find how a nucleus is formed from free protons and neutrons.

As we know that a nucleus consists of protons and neutrons, therefore it may be expected that the mass of the nucleus is equal to the total mass of its protons and neutrons. But actually the nuclear mass M is found to be always less than the total mass of its constituent nucleons. In other words, if a certain number of protons and neutrons are brought together to form a nucleus, the mass of the nucleus is less than the total mass of the nucleons in Free State.

This difference between the sum of the masses of the nucleons constituting a nucleus and the rest mass of the nucleus is known as **mass defect.**

Mass defect $(\Delta m) = (z\ m_p + (A- z)\ m_n - M)$

Where

z is atomic number which is equal to the number of protons,

A is the number of nucleons,

m_p is the mass of a proton,

m_n is the mass of a neutron,

M is the mass of the nucleus,

Video for binding energy:-

Example

Find the mass defect for ${}_8O^{16}$, given that

Mass of a neutron = 1.00866 u

Mass of a proton = 1.00727 u

Mass of oxygen nucleus = 15.99053u

Solution

Oxygen nucleus has 8 protons and 8 neutrons

Mass of 8 neutrons = 8× 1.00866 u = 8.06928 u

Mass of 8 protons = 8×1.00727 u = 8.05816 u

Total mass of 8 neutrons and 8 protons= 16.12744 u

Mass defect = 16.12744 - 15.99053 = 0.13691 u

Nuclear Binding Energy

In the above example, **we have seen that the mass of the oxygen nucleus is less than the total mass of its constituents by 0.13691u.** This means the equivalent energy of the oxygen nucleus is less than that of the sum of the equivalent energies of its constituents.

So if one wants to break the oxygen nucleus into 8 protons and 8 neutrons, extra energy $(\Delta m)c^2$ has to be supplied. Or if a certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass , an energy,

$E = (\Delta m)c^2$, will be released.

This energy released is called the **binding energy.** Thus the binding energy of a nucleus may be defined as t**he energy required to break up a nucleus into its constituent nucleons and to separate them to such a large distance that they may not interact with each other**.

OR

The binding energy may also be defined as **the surplus energy which the nucleons give up by virtue of their attractions when they become bound together to form a nucleus**.

The expression for binding energy is

$B.E = (\Delta m)c^2$, where Δm is a mass defect.

$\Delta m = (z\, m_p + (A - z)\, m_n - M)$

A more useful measure of the binding between the constituents of a nucleus is in terms of binding energy per nucleon, which is defined as "the average energy is required to extract one nucleon from the nucleus".

It can be calculated by using the formula

Binding energy per nucleon = $(\Delta m)c^2 / A$,where A is the number of nucleons in the nucleus

Example

Calculate the binding energy of an alpha particle in MeV, Given that

m_p (the mass of a proton) = 1.007825u

m_n (the mass of a neutron) = 1.008665u

M (the mass of He nucleus) = 4.002800u

Solution

Mass of 2 protons = 2×1.007825 u = 2.015650u

Mass of 2 neutrons = 2×1.008665u = 2.017330 u

Total mass of nucleons = 2.015650u + 2.017330 u = 4. 032980u

Mass defect (Δm) = 4. 032980 u - 4.002800u = 0.030180u

Binding energy = $(\Delta m)c^2$

= $0.030180\ c^2\ (931/\ c^2)$ MeV (using 1u= $(931/\ c^2)$ Mev)

= 28.097 MeV

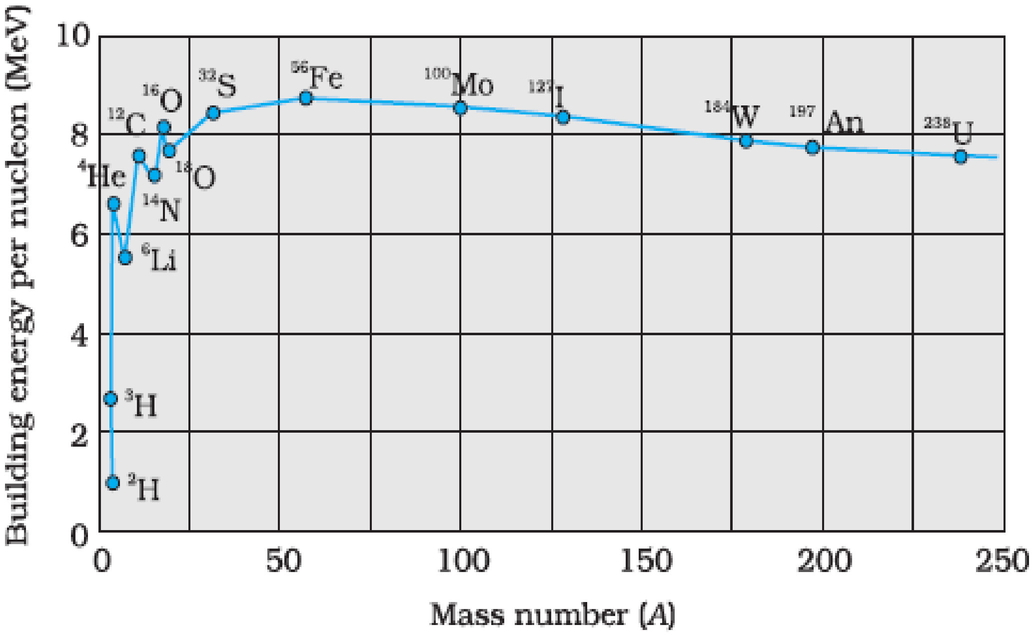
For video on mass defect and binding energy:-

https://www.khanacademy.org/science/physics/quantum-physics/v/mass-defect-and-binding-energy

Nuclear force:- https://www.youtube.com/watch?v=sSjrXZS0xGs

Binding Energy per Nucleon Versus Mass Number(A)Curve

The value of binding energy per nucleon of a nucleus gives a measure of the stability of that nucleus. Greater is the binding energy per nucleon of a nucleus, more stable is the nucleus. The variation of **binding energy per nucleon with the mass number A** is as shown in the graph below.



Observations

- B.E /nucleon is small for light nuclei like $_1H^1$. $_1H^2$ and $_1H^3$.

- In the mass number range 2 to 20, there are well defined maxima for ${}_2He^4$, ${}_6C^{12}$ and ${}_8O^{16}$, indicating the higher stability of these nuclei as compared to their neighboring ones. The minima corresponding to low stability, occur for ${}_3Li^6$, ${}_5B^{10}$, ${}_7N^{14}$
- The curve has broad maxima close to the value 8.5 MeV /nucleon, in the mass number range from 40 to 120. It has a peak value of 8.8 MeV/nucleon for ${}_{26}Fe^{56}$
- E_{bn} is lower for both light nuclei (A<30) and heavy nuclei (A>170).
- As the mass number further increases, the B.E. per nucleon shows a gradual decrease and drops to 7.6MeV/nucleon for ${}_{92}U^{238}$

Conclusions

We can draw the following conclusions from the above observations

- **The force is attractive and sufficiently strong to produce a binding energy of a few Mev per nucleon.**
- If two very light nuclei say, for A < 10, combine to form a heavier nucleus. The binding energy per nucleon of the fused heavier nuclei is more than the binding energy per nucleon of the lighter nuclei. This means that the final nuclei are more stable than the initial nuclei. Thus energy will be released in such a process of fusion. **Thus we can conclude that the lighter nuclei will combine and the energy will be released in the process of fusion.**
- The constancy of the binding energy per nucleon in the range 30 <A <170 is a consequence of the fact that the nuclear force is a **short ranged force**. Consider a particular nucleon inside a sufficiently large nucleus. It will be under the influence of only some of its neighbors, which come in the range of the nuclear force. If any other nucleon is at a distance more than the range of the nuclear force from the particular nucleon, it will have no influence on the binding energy of the nucleon under consideration.

  If a nucleon can have a maximum of p neighbors within the range of the nuclear force, its binding energy would be proportional to p. Let the binding energy of the nucleus be pk, where k is a constant having dimensions of energy. If we increase A by adding nucleons, they will not change the binding energy of the nucleons inside. Thus the binding energy per nucleon is nearly constant. The property that a given nucleon can influence only nucleons close to it is also referred to as the saturation **property of the nuclear force.**

- A heavy nucleus say A = 240 has less B.E/nucleon as compared to that of a comparatively lighter nucleus say A= 120. Thus if a heavy nucleus, say A=240 breaks into two nuclei with A=120, the nucleons get more tightly bound. This implies that the energy will be released in the process of breaking up a heavy nucleus into two nuclei of comparable masses. This process of breaking up a heavy nucleus into two nuclei of comparable mass with release of energy is called **nuclear fission.**

Example

If the binding energy per nucleon in Li^7 and He^4 nuclei are respectively 5.6 MeV and 7.06 MeV,Find the energy released in the reaction $Li^7 + p \rightarrow 2\,He^4$

Solution

B. E. of $Li^7 = 5.6 \times 7 = 39.2$ MeV

B. E. of $He^4 = 4\times 7.06 = 28.24$

B. E. of $2He^4 = 2\times 28.24 = 56.48$ MeV

Energy released = (56.48 - 39.2) MeV = 17.28MeV

Summary

- According to Einstein, it is necessary to treat mass as another form of energy. Like any other form of energy, mass – energy can also be converted into other forms of energy, say kinetic energy and vice-versa.
- Einstein's famous mass – energy equivalence relation is

  $$E = mc^2$$

  Where, m is mass and c is the velocity of light in vacuum.
- The difference between the sum of the masses of the nucleons constituting a nucleus and the rest mass of the nucleus is known as **mass defect.**

  Mass defect $(\Delta m) = (z\, m_p + (A- z)\, m_n - M)$

  Where z is charge number which is equal to the number of protons,

  A is the number of nucleons,

  m_p is the mass of a proton,

  m_n is the mass of a neutron,

  M is the mass of the nucleus.

- The **binding energy** of a nucleus may be defined as the energy required to break up a nucleus into its constituent nucleons and to separate them to such a large distance that they may not interact with each other.

  OR

  The binding energy may also be defined as the surplus energy which the nucleons give up by virtue of their attractions when they become bound together to form a nucleus.
- The expression for binding energy is

  B. E = $(\Delta m)c^2$, where Δm is a mass defect.
- Binding energy per nucleon, is defined as "the average energy required to extract one nucleon from the nucleus. It can be calculated by using the formula

  Binding energy per nucleon = $(\Delta m)c^2 / A$, where A is the number of nucleons in the nucleus
- From binding energy versus mass number curve we see that

  B.E /nucleon is small for light nuclei like ${}_1H^1$. ${}_1H^2$ and ${}_1H^3$.

  In the mass number range 2 to 20 , there are well defined maxima for ${}_2He^4$, ${}_6C^{12}$ and ${}_8O^{16}$, indicating the higher stability of these nuclei as compared to their neighboring ones. The minima corresponding to low stability, occur for ${}_3Li^6$, ${}_5B^{10}$, ${}_7N^{14}$

  The curve has a broad maxima close to the value 8.5 Mev /nucleon, in the mass number range from 40 to 120. It has a peak value of 8.8 Mev/nucleon for ${}_{26}Fe^{56}$

  As the mass number further increases, the B.E. per nucleon shows a gradual decrease and drops to 7.6 Mev/nucleon for ${}_{92}U^{238}$
- The force is attractive and sufficiently strong to produce a binding energy of a few MeV per nucleon.
- The lighter nuclei will combine and the energy will be released in the process of fusion of two lighter nuclei of mass number $A<10$, because the B.E./nucleon of the finally formed nuclei is more than that of the lighter nuclei.
- The constancy of the binding energy per nucleon in the range $30<A<170$ is a consequence of the fact that the nuclear force is a **short ranged and saturated force**.
- Energy will be released in the process of breaking up a heavy nucleus into two nuclei of comparable masses because the B.E per nucleon of the lighter nuclei so formed is more than the B.E/nucleon of the heavy nuclei. This process of breaking up a heavy nucleus into two nuclei of comparable mass with release of energy is called **nuclear fission.**